# TRIGONOMETRI

Trigonometri → berasal dari bahasa Yunani

Trigonometri berasal dari dua kata, yaitu *trigono* = berarti segitiga dan *metri* = ilmu ukur

Jadi trigonometri merupakan ilmu ukur segitiga

Perbandingan Trigonometri Suatu Sudut pada Segitiga Siku-siku
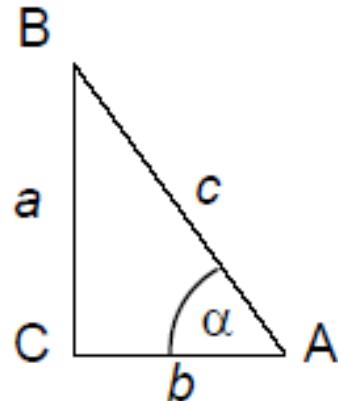
Terhadap sudut α

- Sisi a disebut sisi siku-siku di depan sudut α
- Sisi b disebut sisi siku-siku di dekat (berimpit) sudut α
- Sisi c (sisi miring) disebut hipotenusa

Berdasarkan keterangan di atas, didefinisikan 6 (enam) perbandingan trigonometri terhadap sudut α sebagai berikut:

1. $\sin \alpha = \dfrac{\text{panjang sisi siku - siku di depan sudut A}}{\text{panjang hipotenusa}} = \dfrac{a}{c}$

2. $\cos \alpha = \dfrac{\text{panjang sisi siku - siku di dekat (berimpit) sudut A}}{\text{panjang hipotenusa}} = \dfrac{b}{c}$

3. $\tan \alpha = \dfrac{\text{panjang sisi siku - siku di depan sudut A}}{\text{panjang sisi siku - siku di dekat sudut A}} = \dfrac{a}{b}$

4. $\csc \alpha = \dfrac{\text{panjang hipotenusa}}{\text{panjang sisi siku - siku di depan sudut A}} = \dfrac{c}{a}$

5. $\sec \alpha = \dfrac{\text{panjang hipotenusa}}{\text{panjang sisi siku - siku di dekat sudut A}} = \dfrac{c}{b}$

6. $\cot \alpha = \dfrac{\text{panjang sisi siku - siku di dekat sudut A}}{\text{panjang sisi siku - siku di depan sudut A}} = \dfrac{c}{a}$

Dari perbandingan tersebut dapat pula ditulis rumus:

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} \quad \text{dan} \quad \cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\sec \alpha = \frac{1}{\cos \alpha} \quad \text{dan} \quad \csc \alpha = \frac{1}{\sin \alpha}$$

Nilai Perbandingan Trigonometri untuk Sudut-Sudut Istimewa

α	0°	30°	45°	60°	90°
Sin α	0	1/2	½ √2	½ √3	1
Cos α	1	½ √3	½ √2	1/2	0
Tan α	0	1/3 √3	1	√3	Tak terdefinisi
Cot α	Tak terdefinisi	√3	1	1/3 √3	0

Perbandingan Trigonometri suatu Sudut di Berbagai Kuadran
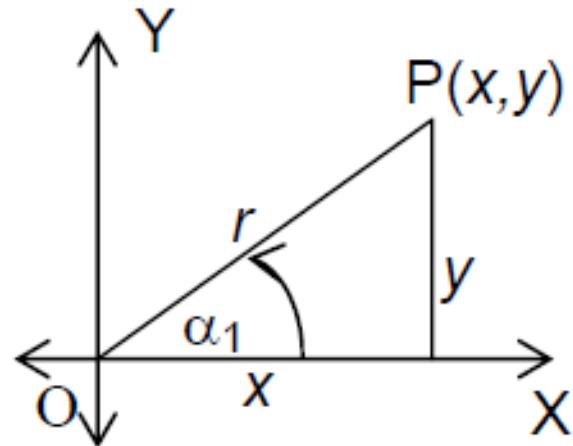
Dimana:

$$OP = \sqrt{x^2 + y^2} = r \ \text{dan} \ r > 0$$

Berdasarkan gambar di atas, diperoleh perbandingan sbb:

1. $\sin \alpha = \dfrac{\text{ordinat } P}{\text{panjang } OP} = \dfrac{y}{r}$

2. $\cos \alpha = \dfrac{\text{absis } P}{\text{panjang } OP} = \dfrac{x}{r}$

3. $\tan \alpha = \dfrac{\text{ordinat } P}{\text{absis } P} = \dfrac{y}{x}$

4. $\csc \alpha = \dfrac{\text{panjang } OP}{\text{ordinat } P} = \dfrac{r}{y}$

5. $\sec \alpha = \dfrac{\text{panjang } OP}{\text{absis } P} = \dfrac{r}{x}$

6. $\cot \alpha = \dfrac{\text{absis } P}{\text{ordinat } P} = \dfrac{x}{y}$

Dengan memutar garis OP diperoleh gambar sbb:
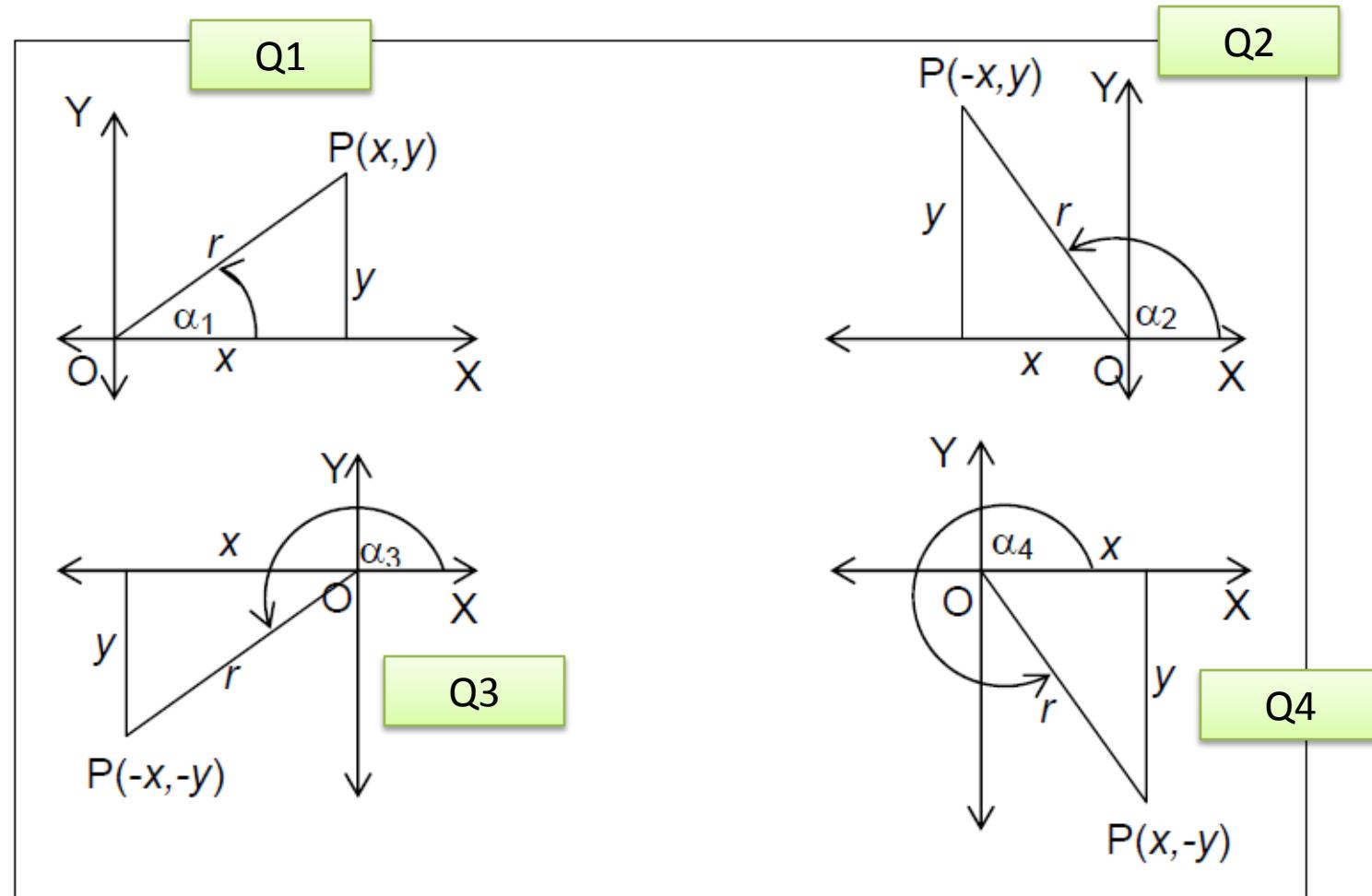
Titik P diberbagai kuadran

Tabel tanda nilai keenam perbandingan trigonometri di tiap kuadran:

Perbandingan Trigonometri	Kuadran			
	I	II	III	IV
Sin	+	+	-	-
Cos	+	-	-	+
Tan	+	-	+	-
Cosec	+	+	-	-
Sec	+	-	-	+
Cot	+	-	+	-

Rumus Perbandingan Trigonometri Sudut yang Berelasi

Perbandingan trigonometri untuk sudut α dengan (90° - α)
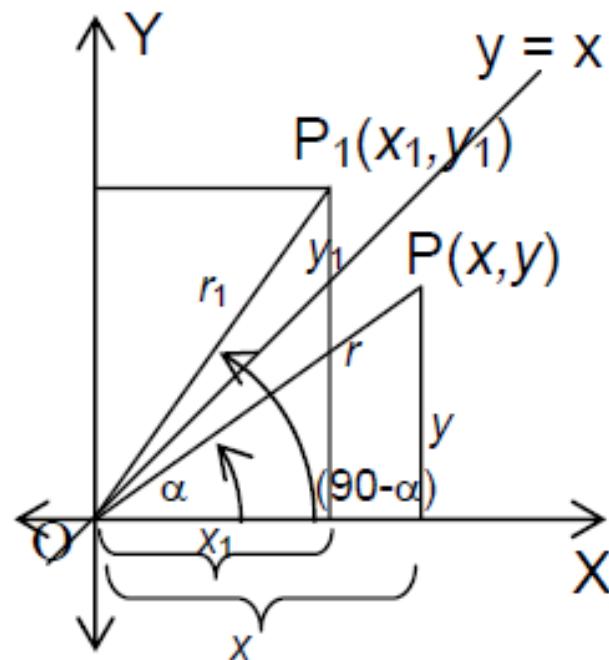
Dari pencerminan garis y = x diperoleh:

a. $\sin(90° - \alpha) = \dfrac{y_1}{r_1} = \dfrac{x}{r} = \cos\alpha$

b. $\cos(90° - \alpha) = \dfrac{x_1}{r_1} = \dfrac{y}{r} = \sin\alpha$

c. $\tan(90° - \alpha) = \dfrac{y_1}{x_1} = \dfrac{x}{y} = \cot\alpha$

- Dari perhitungan tersebut maka rumus perbandingan trigonometri sudut α dengan (90° - α) dapat dituliskan sebagai berikut:

Sin (90° - α) = cos α

cosec (90° - α) = sec α

Cos (90° - α) = sin α

Sec (90° - α) = cosec α

Tan (90° - α) = cot α

Cot (90° - α) = Tan α

Perbandingan trigonometri untuk sudut α° dengan (180° - α)
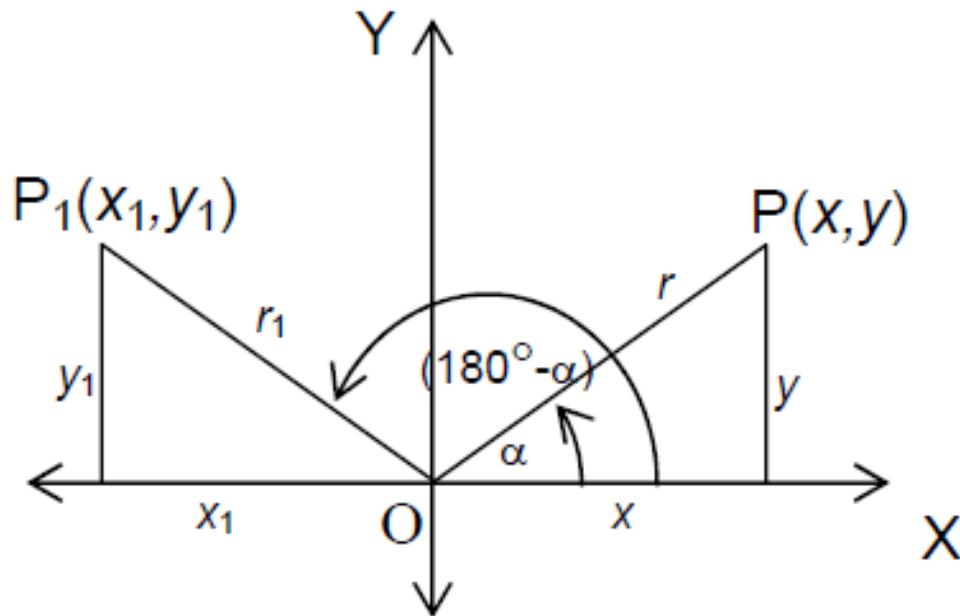
Akibat pencerminan terhadap sumbu Y diperoleh:

a. $\sin(180° - \alpha) = \dfrac{y_1}{r_1} = \dfrac{y}{r} = \sin\alpha$

b. $\cos(180° - \alpha) = \dfrac{x_1}{r_1} = \dfrac{-x}{r} = -\cos\alpha$

c. $\tan(180° - \alpha) = \dfrac{y_1}{x_1} = \dfrac{y}{-x} = -\tan\alpha$

Dari hubungan di atas diperoleh rumus:

$\sin(180° - \alpha) = \sin\alpha$

$\cos(180° - \alpha) = -\cos\alpha$

$\tan(180° - \alpha) = -\tan\alpha$

$\operatorname{cosec}(180° - \alpha) = \operatorname{cosec}\alpha$
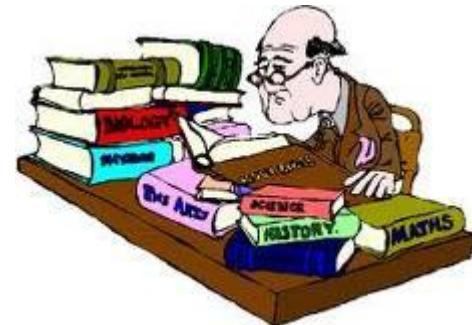
$\sec(180° - \alpha) = -\sec\alpha$

$\cot(180° - \alpha) = -\cot\alpha$

Perbandingan trigonometri untuk sudut α° dengan (180° + α)
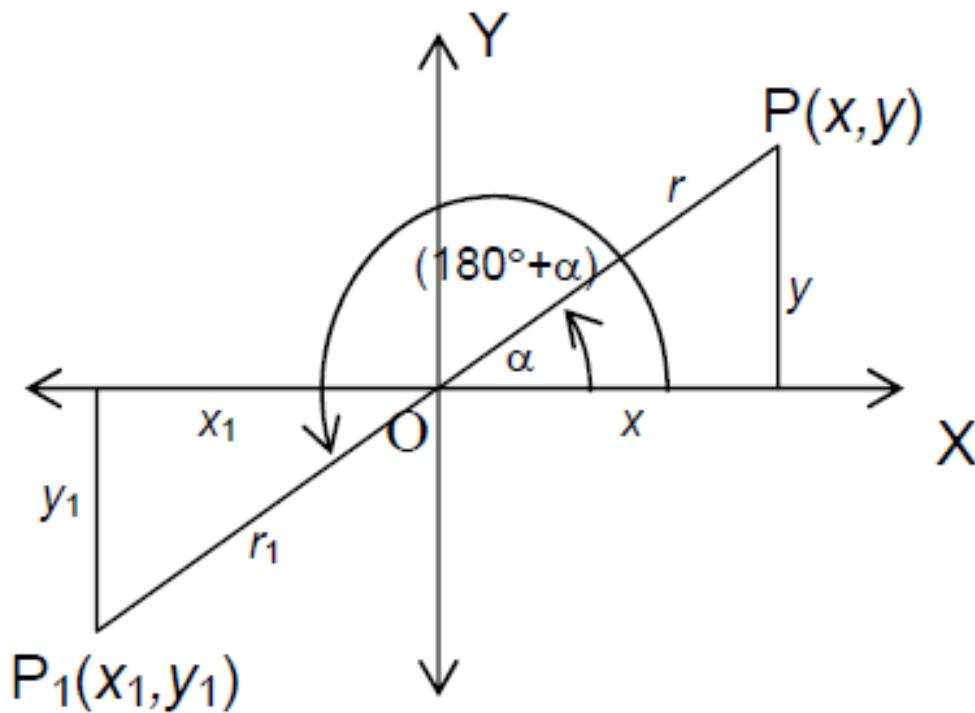
Akibat pencerminan terhadap garis *y = −x* diperoleh:

a. $\sin(180° + \alpha) = \dfrac{y_1}{r_1} = \dfrac{-y}{r} = -\sin \alpha$

b. $\cos(180° + \alpha) = \dfrac{x_1}{r_1} = \dfrac{-x}{r} = -\cos \alpha$

c. $\tan(180° + \alpha) = \dfrac{y_1}{x_1} = \dfrac{-y}{-x} = \dfrac{y}{x} = \tan \alpha$

Dari hubungan di atas diperoleh rumus:

$\mathrm{Sin}\ (180° + \alpha) = -\sin \alpha$

$\mathrm{Cos}\ (180° + \alpha) = -\cos \alpha$

$\mathrm{Tan}\ (180° + \alpha) = \tan \alpha$

$\mathrm{cosec}\ (180° + \alpha) = -\mathrm{cosec}\ \alpha$

$\mathrm{Sec}\ (180° + \alpha) = -\sec \alpha$

$\mathrm{Cot}\ (180° + \alpha) = \cot \alpha$

Perbandingan trigonometri untuk sudut α dengan (- α)
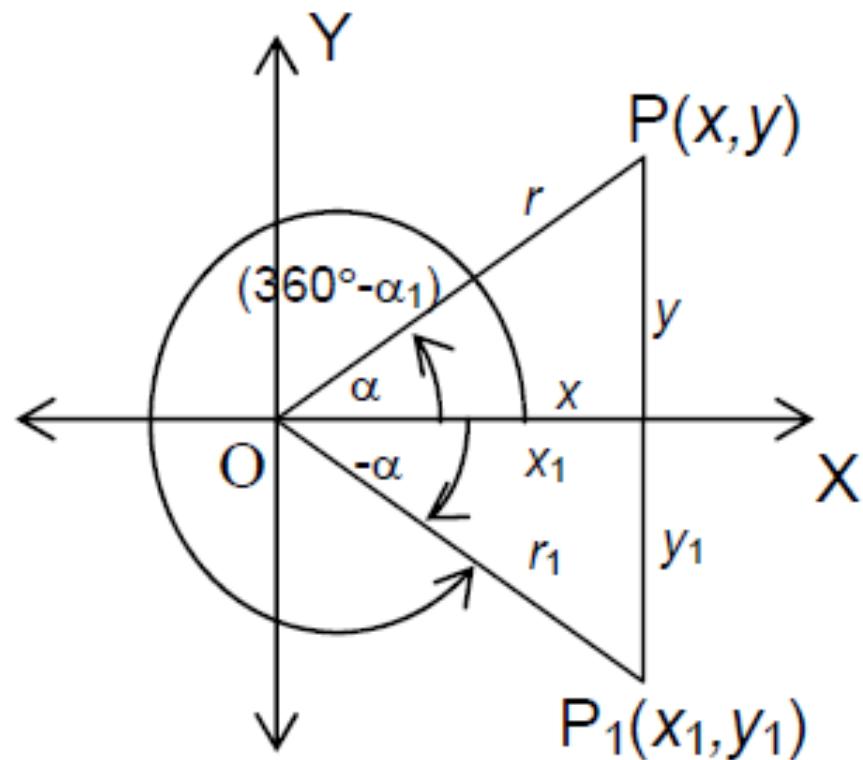
akibat pencerminan terhadap sumbu *x,* diperoleh :

a. $\sin(-\alpha) = \dfrac{y_1}{r_1} = \dfrac{-y}{r} = -\sin \alpha$

b. $\cos(-\alpha) = \dfrac{x_1}{r_1} = \dfrac{x}{r} = \cos \alpha$

c. $\tan(-\alpha) = \dfrac{y_1}{x_1} = \dfrac{-y}{x} = -\tan \alpha$

Dari hubungan di atas diperoleh rumus:

Sin (- α) = - sin α

Cos (- α) = cos α

Tan (- α) = - tan α

cosec (- α) = - cosec α

Sec (- α) = sec α

Cot (- α) = - cot α

Menentukan Koordinat kartesius dan Koordinat Kutub
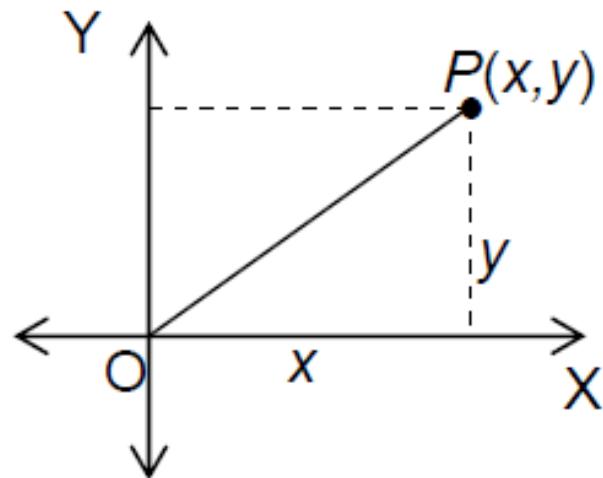
Koordinat kartesius
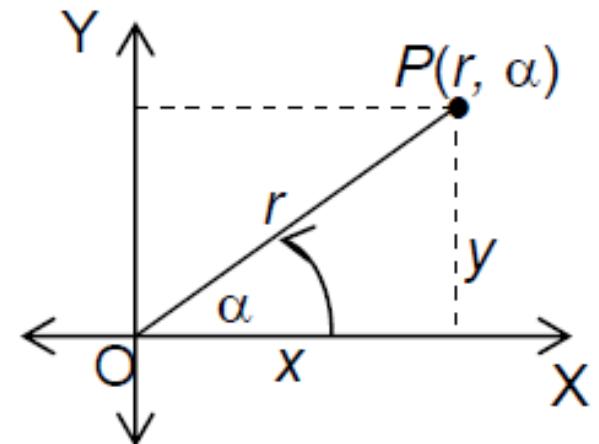
Koordinat kutub

- Jika koordinat kutub titik *P(r, α) diketahui, koordinat kartesius* dapat dicari dengan hubungan:

$$\cos \alpha = \frac{x}{r} \rightarrow x = r \cos \alpha$$

$$\sin \alpha = \frac{y}{r} \rightarrow y = r \sin \alpha$$

- jika koordinat kartesius titik *P(x,y) diketahui, koordinat kutub titik P(r, α)* dapat dicari dengan hubungan:

ket: arc tan adalah invers dari tan

$$r = \sqrt{x^2 + y^2}$$

$$\tan \alpha = \frac{y}{x} \rightarrow \alpha = \arctan \frac{y}{x}$$

Identitas Trigonometri
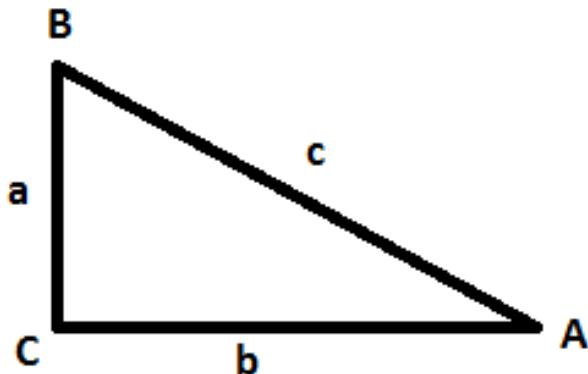
$$a^2 + b^2 = c^2$$

$$\underline{\hspace{4cm}} \quad :c^2$$

$$a^2/c^2 + b^2/c^2 = 1$$

$$(a/c)^2 + (b/c)^2 = 1$$

Karena:

Sin A = a/c dan cos A = b/c

Maka:

$(\sin A)^2 + (\cos A)^2 = 1$

$\sin^2 A + \cos^2 A = 1$

Jika:

$$\sin^2 A + \cos^2 A = 1$$

_____ $:\sin^2 A$

$$\sin^2 A / \sin^2 A + \cos^2 A / \sin^2 A = 1/\sin^2 A$$

$1 + \cot^2 A = \csc^2 A$

Jika:

$$\sin^2 A + \cos^2 A = 1$$

_____ $:\cos^2 A$

$$\sin^2 A / \cos^2 A + \cos^2 A / \cos^2 A = 1/\cos^2 A$$

$\tan^2 A + 1 = \sec^2 A$

Aturan Sinus

sin β = T/A   sin α = T/B

T   = A sin β   T   = B sin α

Jadi→ A $\underline{\sin β}$ = B $\underline{\sin α}$

$\qquad$ sin β $\qquad$ sin β

$\qquad$ A = B $\underline{\sin α}$

$\qquad\qquad\quad$ sin β
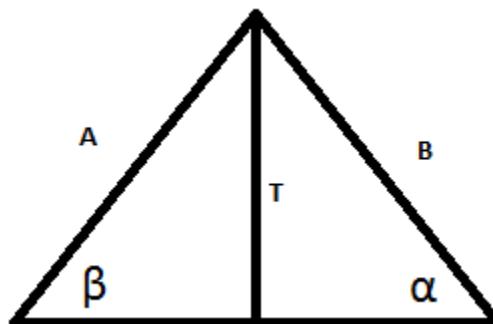
$$\frac{A}{\sin \alpha} = B \frac{\sin \alpha}{\sin \beta} \cdot \frac{1}{\sin \alpha}$$

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta}$$

Jika ditambah sudut γ maka persamaan manjadi:
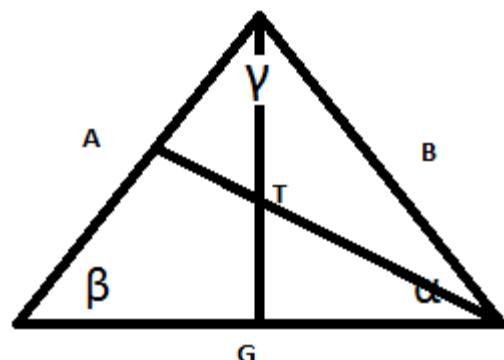
$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{G}{\sin \gamma}$$

Aturan Cosinus
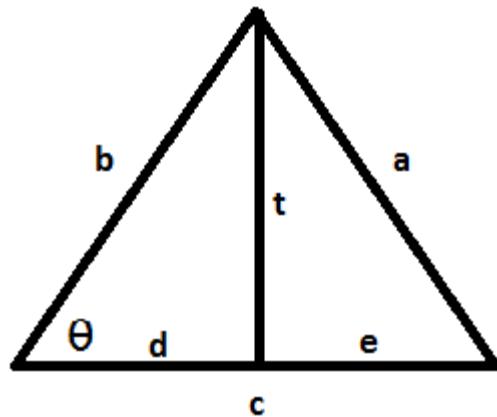
$\cos \theta = d/b$

$d \quad = b \cos \theta$

$e \quad = c - d$

$e \quad = c - b \cos \theta$

$t/b = \sin \theta$

$t \quad = b \sin \theta$

$a^2 \quad = t^2 + e^2$

$a^2 \quad = (b \sin \theta)^2 + (c - b \cos \theta)^2$

$a^2 \quad = b^2 \sin^2 \theta + c^2 - 2bc \cos \theta + b^2 \cos^2 \theta$

$$a^2 = b^2 \sin^2 \theta + c^2 - 2bc \cos \theta + b^2 \cos^2 \theta$$

$$a^2 = b^2 \sin^2 \theta + b^2 \cos^2 \theta + c^2 - 2bc \cos \theta$$

$$a^2 = b^2 (\sin^2 \theta + \cos^2 \theta) + c^2 - 2bc \cos \theta$$

$$a^2 = b^2 . 1 + c^2 - 2bc \cos \theta$$

$$\mathbf{a^2 = b^2 + c^2 - 2bc \cos \theta}$$

Rumus-rumus Trigonometri untuk Jumlah dan Selisih Dua Sudut

- $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Rumus Trigonometri Sudut Rangkap

1. $\sin 2\alpha = \sin (\alpha + \alpha) = \sin \alpha \cos \alpha + \cos \alpha \sin \alpha = 2 \sin\alpha \cos\alpha$

   $\sin 2\alpha = 2 \sin\alpha \cos\alpha$

2. $\cos 2\alpha = \cos (\alpha + \alpha) = \cos \alpha \cos \alpha - \sin \alpha \sin \alpha = \cos^2\alpha - \sin^2\alpha$

   $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

3. $$\tan 2\alpha = \tan (\alpha + \alpha) = \frac{\tan \alpha + \tan \alpha}{1 - \tan \alpha \tan \alpha} = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

Mengubah Rumus Perkalian ke rumus Penjumlahan/Pengurangan

1. Dari rumus cosinus untuk jumlah dan selisih 2 sudut diperoleh:

$$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\overline{\cos (\alpha + \beta) + \cos (\alpha - \beta) = 2 \cos \alpha \cos \beta}^{+}$$

Jadi
$$\boxed{\cos (\alpha + \beta) + \cos (\alpha - \beta) = 2 \cos \alpha \cos \beta}$$

$$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\overline{\cos (\alpha + \beta) - \cos (\alpha - \beta) = -2 \sin \alpha \sin \beta} \quad -$$

Jadi

$$\boxed{\cos (\alpha + \beta) - \cos (\alpha - \beta) = -2 \sin \alpha \sin \beta}$$

2. Dari rumus sinus untuk jumlah dan selisih 2 sudut diperoleh:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\overline{\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta}\quad +$$

Jadi: **$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta$**

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\overline{} \; -$$

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos \alpha \sin \beta$$

Jadi: **sin (α + β) − sin (α − β) = 2 cos α sin β**